Assessing Volatility Persistence in Fractional Heston Models with Self-exciting Jumps

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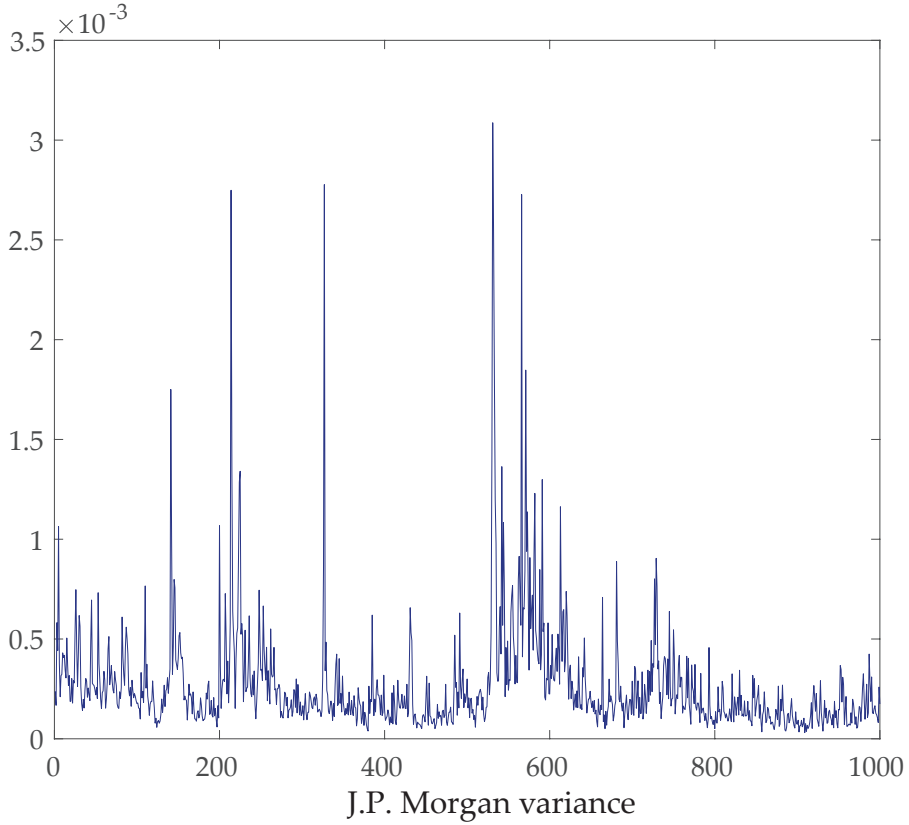
Volatility dynamics

- complex, unobserved and requires specific methods to model & forecast
- mainly driven by two stylized facts:
  - persistence
  - presence of jumps in asset prices



Volatility dynamics and the theoretical literature

- Long-range dependence in volatility
  - ... Comte, F., Renault, E., (1998). Long memory in continuous-time stochastic volatility models. *Mathematical Finance*.
  - ... Comte, F., Coutin, L., Renault, E., (2012). Affine fractional stochastic volatility models. *Annals of Finance*.
- Volatility modeling in presence of jumps
  - ... Aït-Sahalia, Y. and Jacod, J., (2012). Analyzing the spectrum of asset returns: jump and volatility components in high frequency data. *JEL*.
  - ... Aït-Sahalia, Y., Cacho-Diaz, J., Laeven, R.J.A., (2015). Modeling financial contagion using mutually exciting jump processes. *JFE*.
- Few empirical papers merge these two stylized facts
  - e.g. Boudt et al., (2011). Robust estimation of intraweek periodicity in volatility and jump detection. *Journal of Empirical Finance*.
- But no theoretical paper tackles these issues at the same time

Objective

- Propose a realistic continuous-time model, which accounts for both general forms of jumps and persistence
- Study volatility dynamics in this theoretical framework
- Empirically analyze volatility forecasting in this context

Theoretical framework

Continuous-time model:

$$dS(\tau) = \mu(\mu_S, \lambda^\star(\tau), \zeta(\tau))S(\tau)d\tau + \sigma(\tau)S(\tau)dW_S(\tau) + (\zeta_S - 1)S(\tau)dN(\tau)$$

$$\sigma^2(\tau) = \eta + \int_{-a}^{\tau} \frac{(\tau - s)^\delta}{\Gamma(1+\delta)} dV(s)$$

$$dV(\tau) = \kappa(\theta - V(\tau))d\tau + \varsigma\sqrt{V(\tau)}dW_\sigma(\tau)$$

$$\lambda^\star(\tau) = \bar{\lambda} + \int_{-\infty}^{\tau} \nu(\tau - u)dN(u)$$

with

- $S(\tau)$ the true price process of a given asset at a continuous time τ
- μ the compensated drift term

Theoretical framework

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with

- $\sigma^2(\tau)$ instantaneous fractionally integrated variance process
- $\delta \in (0, 1/2)$, the long-memory parameter
- $a \geq 0$
- $W_S(\tau)$ and $W_\sigma(\tau)$ two independent standard Brownian motions
- Feller condition: $\theta, \kappa > 0$, $\kappa\theta \geq \varsigma^2/2$

Theoretical framework

Continuous-time model:

$$dS(\tau) = \mu(\mu_S, \lambda^\star(\tau), \zeta(\tau))S(\tau)d\tau + \sigma(\tau)S(\tau)dW_S(\tau) + (\zeta_S - 1)S(\tau)dN(\tau)$$

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$$\lambda^\star(\tau) = \bar{\lambda} + \int_{-\infty}^{\tau} \nu(\tau - u)dN(u)$$

with

- $(\zeta(\tau) - 1)dN(\tau)$ a finite activity jump process
- $N(\tau)$ an Hawkes process with time varying intensity parameter $\lambda^*(\tau)$
- $\zeta(\tau)$ i.i.d. log-normal random variable

Quadratic variation of prices

- Solution of the SDE

$$p(\tau) = p(0) + \int_0^\tau \left(\check{\mu}(u) - \frac{1}{2}\sigma^2(u)\right)du + \int_0^\tau \sigma(u)\, dW_S(u) + \int_0^\tau \tilde{\zeta}\, dN(u),$$

with $\tilde{\zeta} = \ln\left(\zeta(u)\right) \sim$ i.i.d. $\mathcal{N}(\tilde{\mu}, \tilde{\sigma})$.

- Quadratic variation

$$\sum_{j=1}^{M} \left(p(\tau_{(t-1)M+j+1}) - p(\tau_{(t-1)M+j})\right)^2 \xrightarrow[M\to\infty]{p} [p,p]_t = QV(t)$$

$$QV(t) = \underbrace{\int_{t-1}^{t} \sigma_\tau^2 d\tau}_{IV_t \equiv \mathscr{C}(t)} + \underbrace{\sum_{N(t-1)+1}^{N(t)} \tilde{\xi}(\tau_i)^2}_{JV_t \equiv \mathscr{D}(t)}$$

Quadratic variation of prices

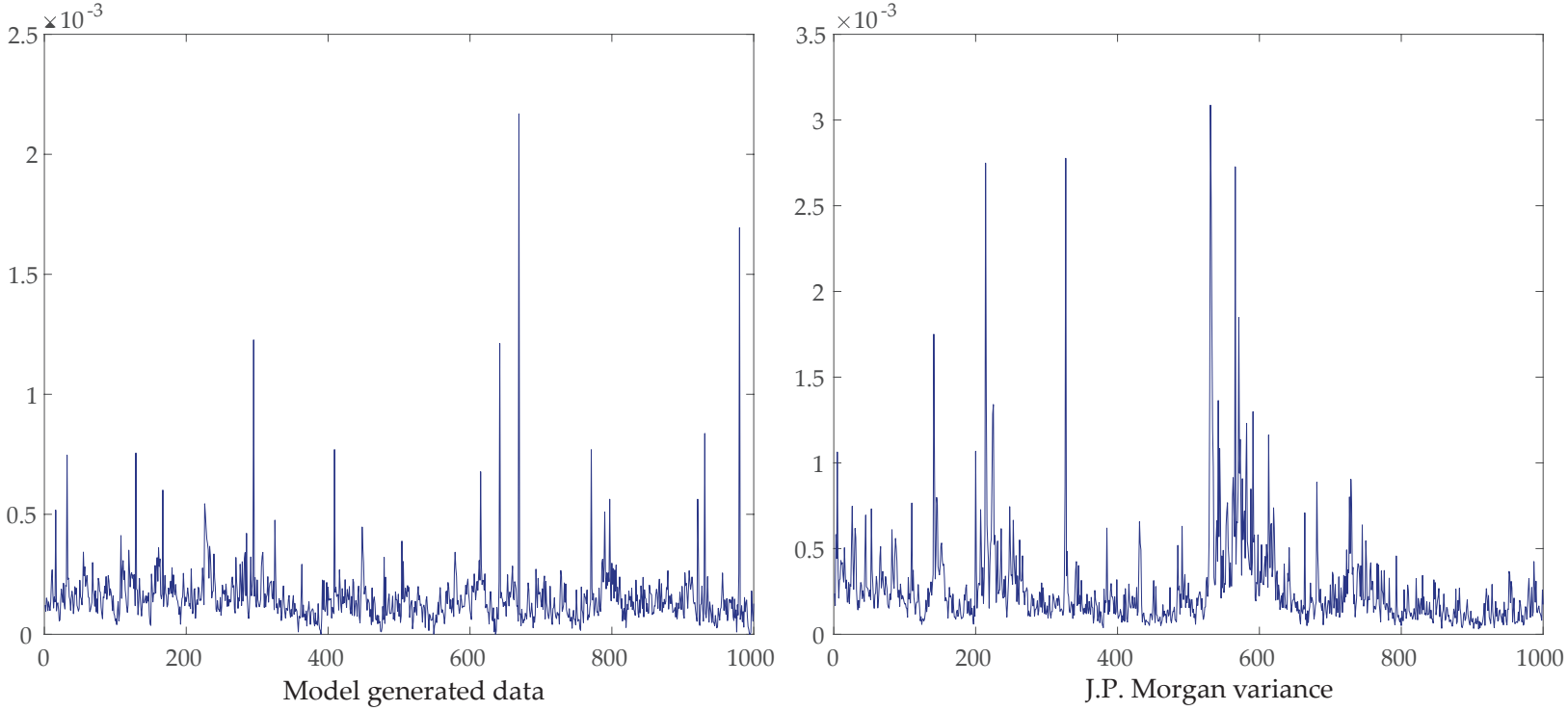


Figure: The variance generated by the fractional Heston model with jumps (left) was obtained by setting $\delta = 0.4$, $\kappa = 5$, $\theta = 0.1$, $\varsigma = 0.9$, $\eta = 0.3$, $\bar{\lambda} = 0.001$, $\alpha = 0.8$, $\beta = 1$, $\tilde{\mu} = 0.005$, $\tilde{\sigma} = 0.15$, and $\sigma_\epsilon = 0.01$. The daily realized variance of JPMorgan (right) is based on 5-minutes returns.

Time domain long memory analysis

- ACF decomposition

$$ACF(h) = \frac{c_{\mathscr{C}}(h)}{\mathbb{V}(\mathscr{C}) + \mathbb{V}(\mathscr{D})} + \frac{c_{\mathscr{D}}(h)}{\mathbb{V}(\mathscr{C}) + \mathbb{V}(\mathscr{D})}$$

Proposition (1)

Let $\delta \in (0, 1/2)$. The autocovariance function of $\mathscr{C}(t)$ takes the form

$$\begin{aligned} c_{\mathscr{C}}(h) &= \int_0^1 \int_0^1 c_{\sigma^2}(u + h - s)\, ds\, du \\ &= \frac{\mathcal{C}}{2\delta(2\delta+1)} \int_{-\infty}^{+\infty} \left(|v-1|^{2\delta+1} + |v+1|^{2\delta+1} - 2\,|v|^{2\delta+1} \right) e^{-\kappa|v-h|}\, dv \end{aligned}$$

with $\mathcal{C} = \frac{\varsigma^2\theta}{2\kappa} \frac{\Gamma(1-2\delta)}{\Gamma(1-\delta)\Gamma(\delta)}$, $h \in [0, \infty)$,

$$c_{\sigma^2}(h) = \mathcal{C} \int_{-\infty}^{+\infty} |h+v|^{2\delta-1}\, e^{-k|v|}\, dv,$$

Asymptotically, as $h \to \infty$

$$c_{\mathscr{C}}(h) \sim \frac{2\mathcal{C}}{\kappa}\, h^{2\delta-1}.$$

Time domain long memory analysis

Assumption (1)

We suppose that the excitation function is exponential $\nu(t) = \alpha e^{-\beta t}$, with $\alpha \geq 0$, $\beta \geq 0$ and $\alpha/\beta < 1$ so that the intensity process is stationary.

Note: If $\alpha = 0$, $N(t)$ is a standard Poisson point process

Proposition (2)

Then, the autocovariance function of $\mathscr{D}(t)$ is

$$c_{\mathscr{D}}(h) = \overline{\lambda^\star}(1-h)\mathbb{V}(\tilde{\zeta}^2)\mathbb{1}_{(h<1)} + \mathbb{E}(\tilde{\zeta}^2)^2 c_{\tilde{N}_t}(h), \text{ with}$$

$$c_{\tilde{N}_t}(h) = (1-h)\frac{\beta^3\bar{\lambda}}{(\beta-\alpha)^3}\mathbb{1}_{(h<1)} + \frac{\alpha\beta\bar{\lambda}(2\beta-\alpha)}{2(\beta-\alpha)^4}\left(e^{-(\beta-\alpha)(h+1)} + e^{-(\beta-\alpha)(h-1)} - 2e^{-(\beta-\alpha)h}\right),$$

Time domain long memory analysis

Proposition (2, continued)

As a particular case, for $h = 0$, we obtain

$$\mathbb{V}\left(\mathscr{D}\right) = \overline{\lambda^\star}\mathbb{E}(\tilde{\zeta}^4) + \overline{\lambda^\star}\left(\mathbb{E}(\tilde{\zeta}^2)\right)^2 \frac{\alpha(2\beta-\alpha)}{(\beta-\alpha)^2}\left(1 - \frac{1-e^{-(\beta-\alpha)}}{\beta-\alpha}\right),$$

$$\overline{\lambda^\star} = \bar{\lambda}\beta/(\beta-\alpha)$$

Asymptotically, as $h \to \infty$

$$c_{\mathscr{D}}(h) \sim \mathcal{K}e^{-(\beta-\alpha)h},$$

with

$$\mathcal{K} = \mathbb{E}(\tilde{\zeta}^2)^2 \frac{\alpha\beta\bar{\lambda}(2\beta-\alpha)}{2(\beta-\alpha)^4}\left(e^{-(\beta-\alpha)} + e^{(\beta-\alpha)} - 2\right).$$

ACF of QV

- The function is analytically intractable $\Rightarrow$ numerical approximation

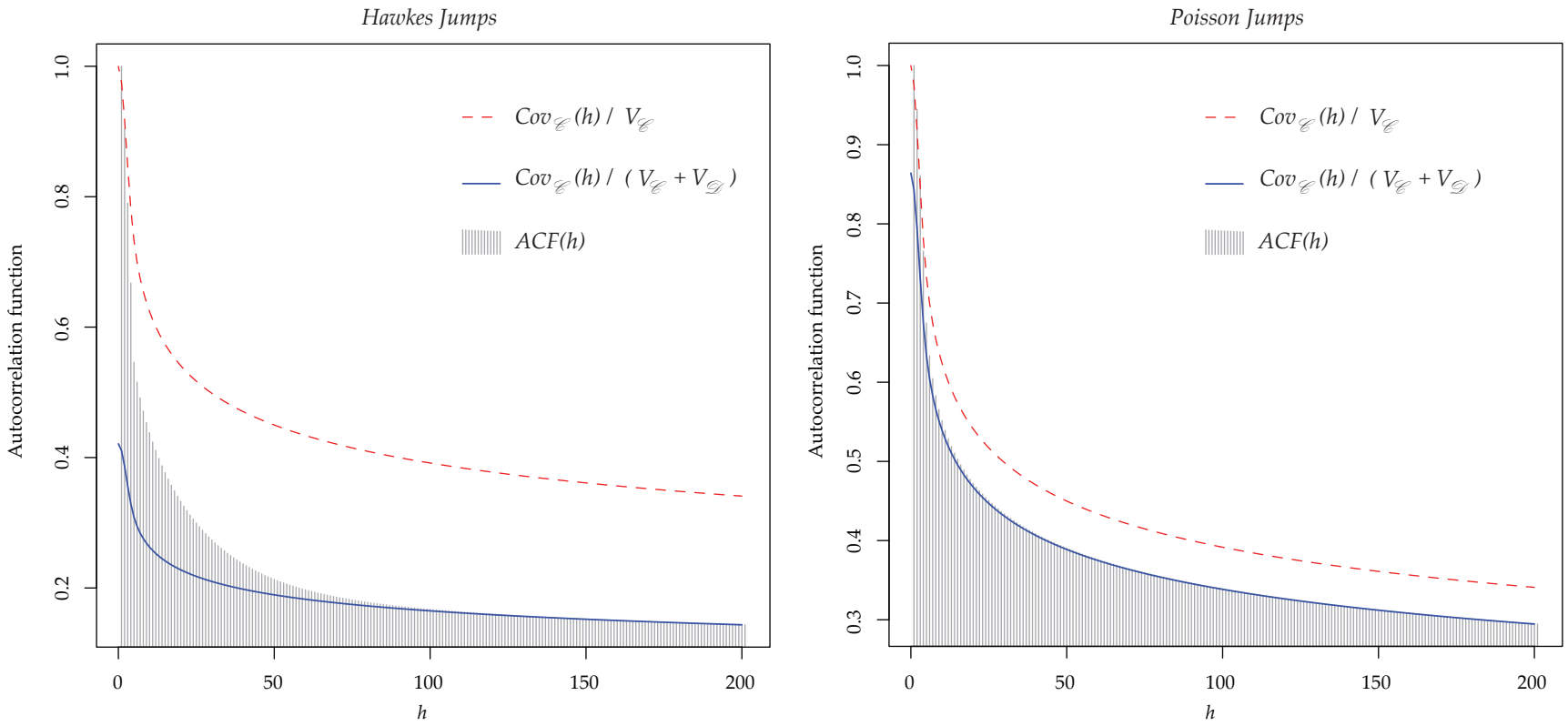


Figure: The plots were obtained by setting $\delta = 0.4$, $\kappa = 5$, $\theta = 0.1$, $\varsigma = 0.9$, $\bar{\lambda} = 0.001$, $\beta = 1$, $\tilde{\mu} = 0.005$, $\tilde{\sigma} = 0.75$ and $\alpha = \{0.8, 0\}$ for the left and right plot, respectively.

Spectral density of QV

Proposition (3)

Let $\omega \in [-\pi, \pi]$, $\delta \in (0, 1/2)$ and assumption 1 hold. We prove that

i) For any ω,

$$f_{\mathscr{C}}(\omega) = \varsigma^2 \theta \frac{|1 - e^{-i\omega}|^2}{\omega^{2\delta+2}(\omega^2 + \kappa^2)}$$

$$f_{\mathscr{D}}(\omega) = \overline{\lambda^\star} \frac{|1 - e^{-i\omega}|^2}{\omega^2} \left(\mathbb{E}(\tilde{\zeta}^4) + \mathbb{E}(\tilde{\zeta}^2)^2 \frac{\alpha(2\beta - \alpha)}{\omega^2 + (\beta - \alpha)^2} \right).$$

ii) As $\omega \to 0$,

$$f_{\mathscr{C}}(\omega) \sim \varphi \omega^{-2\delta}, \text{ with } \varphi = \varsigma^2 \theta / \kappa^2$$

$$f_{\mathscr{D}}(\omega) = \overline{\lambda^\star} \left(\mathbb{V}(\tilde{\zeta}^2) + \mathbb{E}(\tilde{\zeta}^2)^2 \frac{\beta^2}{(\beta - \alpha)^2} \right).$$

Spectral density of QV

It follows that

$$f_{QV}(\omega) = \varsigma^2\theta \frac{|1-e^{-i\omega}|^2}{\omega^{2\delta+2}(\omega^2+\kappa^2)} + \overline{\lambda^\star}\frac{|1-e^{-i\omega}|^2}{\omega^2}\left(\mathbb{E}(\tilde{\zeta}^4) + \mathbb{E}(\tilde{\zeta}^2)^2 \frac{\alpha(2\beta-\alpha)}{\omega^2+(\beta-\alpha)^2}\right),$$

and near zero frequency it takes the form

$$f_{QV}(\omega) \sim \omega^{-2\delta}\varphi(\omega) + \varkappa(\omega), \quad \omega \to 0,$$

with $\varphi(\omega) = \omega^{2\delta} f_{\mathscr{C}}(\omega)$ and $\varkappa(\omega) = f_{\mathscr{D}}(\omega)$.

- In theory $\varphi(\omega)$ and $\varkappa(\omega)$ are constant at zero frequency (φ and $\varkappa$)
- In practice, estimators approximate these terms in the vicinity of the origin and high frequency contamination may occur

Local Whittle-type long memory estimators

- Robust to misspecification of the short-run dynamics

1. Robinson (1995): $f(\omega) \sim \varphi\omega^{-2\delta}$ as $\omega \to 0$, with $\varphi > 0$

$$\hat{\delta}_{LW} = \arg\min_{\delta\in\Theta} R_m(\delta), \quad \omega_j = 2\pi j/T$$

where $R_m(\delta) = \log\hat{\varphi}(\delta) - 2\delta\frac{1}{m}\sum_{j=1}^{m}\log\omega_j, \quad \hat{\varphi}(\delta) = \frac{1}{m}\sum_{j=1}^{m}\omega_j^{2\delta}I(\omega_j)$

2. Hurvich et al., 2005 ($\hat{\delta}_{LWN}$): $f(\omega) \sim \omega^{-2\delta}\varphi + \varkappa$ as $\omega \to 0$, with $\varkappa > 0$
3. Frederiksen et al. 2008 ($\hat{\delta}_{LPWN}$): $f(\omega) \sim \omega^{-2\delta}\tilde{\varphi}(\omega) + \varkappa$
4. Frederiksen et al. 2012 ($\hat{\delta}_{LPWLPN}$): $f(\omega) \sim \omega^{-2\delta}\tilde{\varphi}(\omega) + \tilde{\varkappa}(\omega)$

DGP

Generate a sequence of discretized log-prices

- Discretization step $\Delta\tau = \tau_j - \tau_{j-1} = 1\text{min}$ ($M = 390$) and $T = 1000$ days
- Model parameters:
  - Long Memory: $\delta = 0.4$,
  - $\sigma^2_{\tau_i}$: $\kappa = 5$, $\theta = 0.1$, $\varsigma = 0.9$, $\eta = 0.3$
  - Jump size: $\tilde{\mu} = 0.005$, $\tilde{\sigma} = 0.15$
  - Jump arrival: $\beta = 1$, $(\alpha, \bar{\lambda}) \in \{\underbrace{(0, 0.001)}_{\text{Poisson J}}, \underbrace{(0.5, 0.0005), (0.8, 0.0002)}_{\text{Self-exciting Hawkes J}}\}$

LW estimators of long memory in IV

Unobserved daily integrated variance: $IV_t = \sum_{j=1}^{M} \sigma^2_{(t-1)M+j}$, $t = 1, \ldots, T$

- 1000 replications
- Model parameters are set to $\delta = 0.4$, $\kappa = 5$, $\theta = 0.1$, $\varsigma = 0.9$
- The bandwidth is fixed to $m = \lfloor T^{0.5} \rfloor$ for $\hat{\delta}_{LW}$ and to

$$m = \lfloor T^{0.8} \rfloor$$

for $\hat{\delta}_{LWN}$, $\hat{\delta}_{LPWN}$, and $\hat{\delta}_{LPWLPN}$

$\Rightarrow$ Performances of all estimators are very close

	Bias	Variance	MSE
$\hat{\delta}_{LW}$	-0.0030	0.0139	0.0139
$\hat{\delta}_{LWN}$	0.0444	0.0018	0.0038
$\hat{\delta}_{LPWN}$	-0.0013	0.0054	0.0054
$\hat{\delta}_{LPWLPN}$	0.0448	0.0116	0.0136

LW estimators of long memory in QV

- Observed log-prices

$$p^*_{\tau_i} = p_{\tau_i} + \epsilon_{\tau_i},$$

with $\epsilon_{\tau_i} \sim \mathcal{N}(0, 0.01)$

- Realized measures of variance ($\widehat{QV}$): no optimal sampling frequency
- Measurement error impacts long memory estimates of $\widehat{QV}$:

$$\underbrace{\text{discretization error} + \text{microstructure noise}}_{\text{Rossi and Santucci de Magistris (2014)}} + \text{jumps} \; (+ \text{short-run dynamics})$$

($\hat{\delta}_{LWN}$ of Hurvich et al. 2005 works well)

LW estimators of long memory in QV

	Bias				Variance				MSE			
	$\mathscr{C}_t$	$\mathscr{C}_t + \mathscr{D}_t$			$\mathscr{C}_t$	$\mathscr{C}_t + \mathscr{D}_t$			$\mathscr{C}_t$	$\mathscr{C}_t + \mathscr{D}_t$		
$\hat{\delta}_{LW}$		$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0.8$		$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0.8$		$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0.8$
RV	-0.0121	-0.0210	-0.0267	-0.0677	0.0138	0.0135	0.0133	0.0151	0.0139	0.0140	0.0140	0.0197
BPV	-0.0181	-0.0172	-0.0200	-0.0570	0.0140	0.0136	0.0132	0.0151	0.0143	0.0139	0.0136	0.0183
$medRV$	-0.0191	-0.0195	-0.0199	-0.0570	0.0138	0.0139	0.0128	0.0150	0.0142	0.0143	0.0132	0.0183
$\hat{\delta}_{LWN}$												
RV	0.0147	0.0050	0.0009	-0.0074	0.0034	0.0047	0.0068	0.0135	0.0036	0.0048	0.0068	0.0135
BPV	0.0092	0.0091	0.0043	-0.0061	0.0041	0.0044	0.0052	0.0115	0.0042	0.0045	0.0052	0.0116
$medRV$	0.0077	0.0078	0.0038	-0.0064	0.0045	0.0045	0.0054	0.0114	0.0046	0.0046	0.0055	0.0114
$\hat{\delta}_{LPWN}$												
RV	-0.0037	-0.0072	-0.0082	-0.0195	0.0063	0.0070	0.0079	0.0139	0.0063	0.0071	0.0079	0.0143
BPV	-0.0061	-0.0052	-0.0058	-0.0172	0.0069	0.0067	0.0070	0.0117	0.0069	0.0067	0.0071	0.0120
$medRV$	-0.0059	-0.0063	-0.0074	-0.0163	0.0069	0.0068	0.0075	0.0114	0.0069	0.0069	0.0075	0.0117

Note: The results are based on 1000 replications of $T = 1000$ daily observations based on $M = 390$ intra-daily observations sampled at $\Delta\tau$ =1 minute. The bandwidth is fixed to $m = \lfloor T^{0.5} \rfloor$ for $\hat{\delta}_{LW}$ and to $m = \lfloor T^{0.8} \rfloor$ for $\hat{\delta}_{LWN}$ and $\hat{\delta}_{LPWN}$. $\alpha = 0$ for Poisson jumps and $\alpha > 0$ for Hawkes jumps.

LW estimators of long memory in QV (model with leverage)

	Bias				Variance				MSE			
	$\mathscr{C}_t$	$\mathscr{C}_t + \mathscr{D}_t$			$\mathscr{C}_t$	$\mathscr{C}_t + \mathscr{D}_t$			$\mathscr{C}_t$	$\mathscr{C}_t + \mathscr{D}_t$		
$\hat{\delta}_{LW}$		$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0.8$		$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0.8$		$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0.8$
RV	-0.0371	-0.0467	-0.0543	-0.0971	0.0131	0.0129	0.0133	0.0163	0.0145	0.0151	0.0163	0.0257
BPV	-0.0476	-0.0486	-0.0464	-0.0829	0.0133	0.0130	0.0132	0.0153	0.0155	0.0153	0.0154	0.0221
$medRV$	-0.0522	-0.0535	-0.0511	-0.0831	0.0132	0.0130	0.0132	0.0149	0.0159	0.0158	0.0158	0.0218
$\hat{\delta}_{LWN}$												
RV	-0.0197	-0.0210	-0.0137	-0.0167	0.0118	0.0128	0.0151	0.0258	0.0122	0.0133	0.0153	0.0261
BPV	-0.0183	-0.0176	-0.0096	-0.0147	0.0134	0.0135	0.0135	0.0235	0.0137	0.0138	0.0136	0.0237
$medRV$	-0.0162	-0.0172	-0.0106	-0.0179	0.0139	0.0139	0.0149	0.0202	0.0142	0.0142	0.0150	0.0205
$\hat{\delta}_{LPWN}$												
RV	-0.0132	-0.0177	-0.0153	-0.0287	0.0079	0.0087	0.0108	0.0195	0.0081	0.0090	0.0111	0.0203
BPV	-0.0163	-0.0174	-0.0108	-0.0231	0.0094	0.0095	0.0100	0.0167	0.0097	0.0098	0.0101	0.0172
$medRV$	-0.0158	-0.0183	-0.0131	-0.0253	0.0104	0.0099	0.0111	0.0162	0.0106	0.0102	0.0113	0.0169

Note: The results are based on 1000 replications of $T = 1000$ daily observations based on $M = 390$ intra-daily observations sampled at $\Delta\tau$ =5 minutes. $\rho = -0.3$. The bandwidth is fixed to $m = \lfloor T^{0.5} \rfloor$ for $\hat{\delta}_{LW}$ and to $m = \lfloor T^{0.8} \rfloor$ for $\hat{\delta}_{LWN}$ and $\hat{\delta}_{LPWN}$. $\alpha = 0$ for Poisson jumps and $\alpha > 0$ for Hawkes jumps.

Risk forecasting

- A huge literature in finance focuses on risk prediction
  - Persistence plays a crucial role
  - ... GARCH, ARFIMA, HAR
- As Baillie et al. (2012), we focus on frequency domain forecasting
  1. Filter the long-run component of the log $\widehat{QV}$ with $\hat{\delta}_{LW}$ ($2S - LW$ approach) and $\hat{\delta}_{L(P)WN}$ ($2S - L(P)WN$ approaches)
  2. Estimate the short-run parameters, ϑ, of each resulting process with ARMA$(1,1)$ full-band Whittle framework
  3. Compute iterated forecasts in frequency domain (see Bhansali and Kokoszka, 2002; Hidalgo and Yajima, 2002)

Data

- The dataset runs from January 2010 to June 2016

$\Rightarrow$ 11 largest US financial institutions

- Rolling forecasting scheme (1000 days in-sample, 635 out-of-sample)
- Forecast horizon $h \in \{1, 5, 10, 22\}$ days ahead
- Use robust loss functions (Patton 2011) to assess out-of-sample forecast accuracy
- ARMA(1,1) and log-HAR as competitors

QV forecast comparison

Table: Variance forecast comparison under various robust loss functions

		MSE					*QLIKE*					*AL*				
		ARMA	log-HAR	2S-LW	2S-LWN	2S-LPWN	ARMA	log-HAR	2S-LW	2S-LWN	2S-LPWN	ARMA	log-HAR	2S-LW	2S-LWN	2S-LPWN
$h=1$	*RV*	0.8572	0.8490	0.8438	**0.8433**	**0.8433**	0.1628	0.1609	0.1587	**0.1586**	**0.1586**	0.1608	0.1588	0.1568	**0.1566**	**0.1566**
	RK	1.9424	1.9285	1.9240	**1.9220**	1.9221	0.2081	0.2038	0.2005	**0.2003**	**0.2003**	0.2141	0.2097	0.2069	**0.2066**	**0.2066**
	BV	7.8199	**7.8143**	7.8494	7.8348	7.8432	**0.1269**	0.1306	0.1277	0.1274	0.1274	**0.1514**	0.1553	0.1524	0.1520	0.1521
	medRV	4.7003	**4.6982**	4.7197	4.7091	4.7110	**0.1200**	0.1228	0.1204	0.1202	0.1202	**0.1370**	0.1401	0.1377	0.1374	0.1375
$h=5$	*RV*	1.0445	1.0114	1.0072	**1.0049**	1.0050	0.2544	0.2505	0.2383	**0.2375**	0.2376	0.2469	0.2407	0.2322	**0.2313**	0.2314
	RK	2.1682	2.1247	2.1171	**2.1147**	2.1155	0.3129	0.3080	0.2912	**0.2905**	0.2906	0.3135	0.3053	0.2934	**0.2926**	0.2927
	BV	8.0537	8.0167	8.0094	**8.0055**	8.0061	0.2568	0.2616	0.2444	0.2429	**0.2425**	0.2700	0.2705	0.2585	0.2571	**0.2568**
	medRV	4.9022	4.8708	4.8642	**4.8616**	4.8619	0.2367	0.2422	0.2270	0.2260	**0.2257**	0.2431	0.2443	0.2338	0.2329	**0.2327**
$h=10$	*RV*	1.1075	1.0692	1.0521	1.0496	**1.0494**	0.2925	0.2845	0.2613	**0.2596**	**0.2596**	0.2824	0.2714	0.2539	**0.2524**	**0.2524**
	RK	2.2326	2.1895	2.1673	**2.1654**	2.1659	0.3420	0.3384	0.3100	**0.3089**	0.3090	0.3424	0.3336	0.3119	**0.3109**	0.3110
	BV	8.1588	8.1325	8.1112	**8.1074**	8.1080	0.2960	0.3062	0.2713	0.2694	**0.2692**	0.3062	0.3089	0.2835	0.2817	**0.2816**
	medRV	4.9873	4.9623	4.9431	**4.9408**	4.9412	0.2839	0.2920	0.2583	0.2571	**0.2569**	0.2844	0.2858	0.2619	0.2608	**0.2607**
$h=22$	*RV*	1.1684	1.1321	1.0945	**1.0916**	**1.0916**	0.3117	0.3077	0.2735	0.2727	**0.2726**	0.3045	0.2954	0.2670	0.2661	**0.2660**
	RK	2.3224	2.2872	2.2397	**2.2368**	2.2370	0.3668	0.3731	0.3275	**0.3265**	**0.3265**	0.3697	0.3676	0.3304	**0.3294**	**0.3294**
	BV	8.3381	8.3293	8.2884	8.2864	**8.2851**	0.3057	0.3265	0.2809	0.2806	**0.2793**	0.3198	0.3313	0.2952	0.2947	**0.2936**
	medRV	0.4685	0.4174	0.4129	0.4110	**0.4105**	0.2947	0.3158	0.2714	0.2711	**0.2697**	0.2991	0.3106	0.2764	0.2760	**0.2748**

Note: The bandwidth is fixed to $m = \lfloor T^{0.5} \rfloor$ for $\hat{\delta}_{LW}$ and to $m = \lfloor T^{0.8} \rfloor$ for $\hat{\delta}_{LWN}$ and $\hat{\delta}_{LPWN}$.

Conclusion

- In this paper we have proposed a fractional Heston model with Hawkes-type jump
- We have theoretically derived the autocorrelation and spectral density functions of the QV
- We show that the persistence of QV exhibits a downward bias in presence of jumps
- Robust local Whittle estimators appear as a convenient solution
- We propose a frequency-domain forecasting approach that seems to outperform traditional methods

The end

Thank you for your attention!

LW estimators of long memory in QV

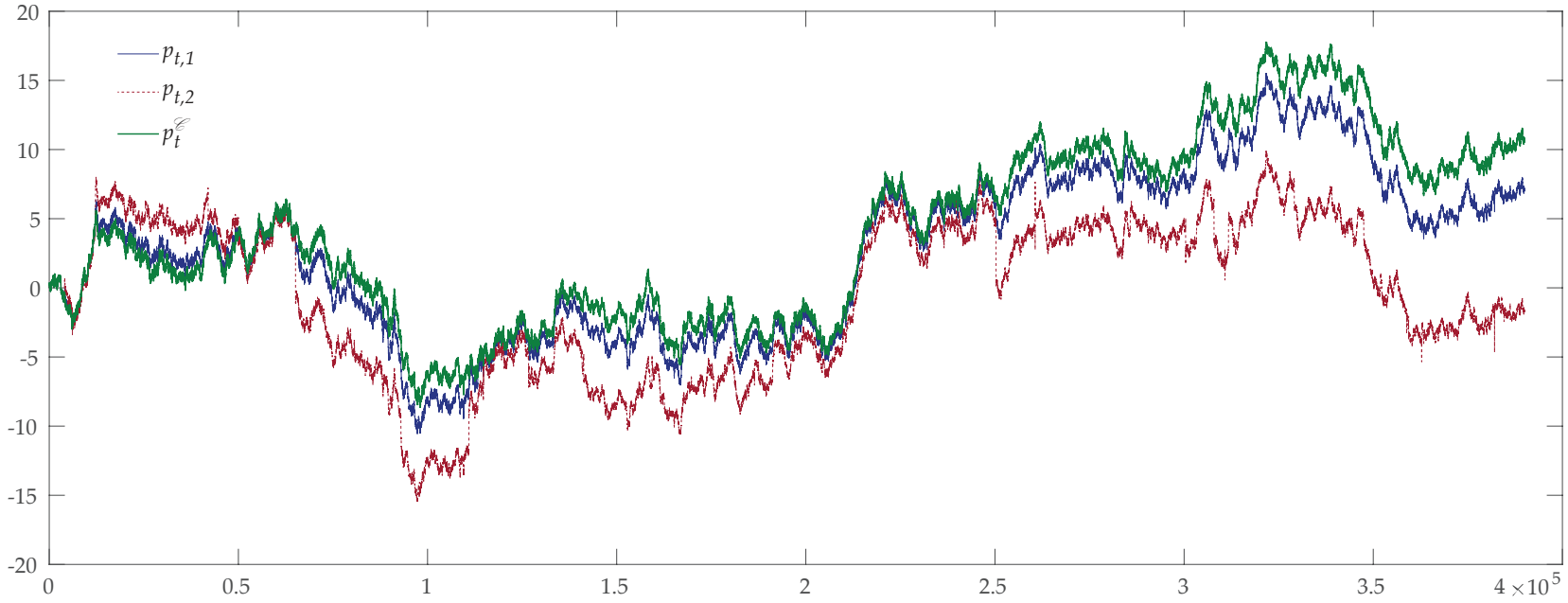


Figure: 1-minute log-prices generated by the fractional Heston model with jumps ($p_{t,1}$ and $p_{t,2}$) and the continuous underlying log-price ($p_t^{\mathscr{C}}$). The plots were obtained by setting $\delta = 0.4$, $\kappa = 5$, $\theta = 0.1$, $\varsigma = 0.9$, $\eta = 0.3$, $\bar{\lambda} = 0.001$, $\alpha = 0.8$, $\beta = 1$, $\tilde{\mu} = 0.005$, and $\sigma_\epsilon = 0.01$. $\tilde{\sigma} = 0.15$ for $p_{t,1}$ and $\tilde{\sigma} = 0.5$ for $p_{t,2}$.

Our frequency-domain forecasting approach for QV

3. One-step and multistep forecasts

$$\log \widehat{QV}^f_{T+1} = -\sum_{u=1}^{M-1} \hat{a}_u \log \widehat{QV}_{T+1-u},$$

$$\log \widehat{QV}^f_{T+h} = -\sum_{u=1}^{h} \hat{a}_u \log \widehat{QV}^f_{T+h-u} - \sum_{u=1}^{M-h-1} \hat{a}_{u+h} \log \widehat{QV}_{T-u},$$

with $M > h$ the optimal number of lags (AIC with penalty $v = 1.2$) and

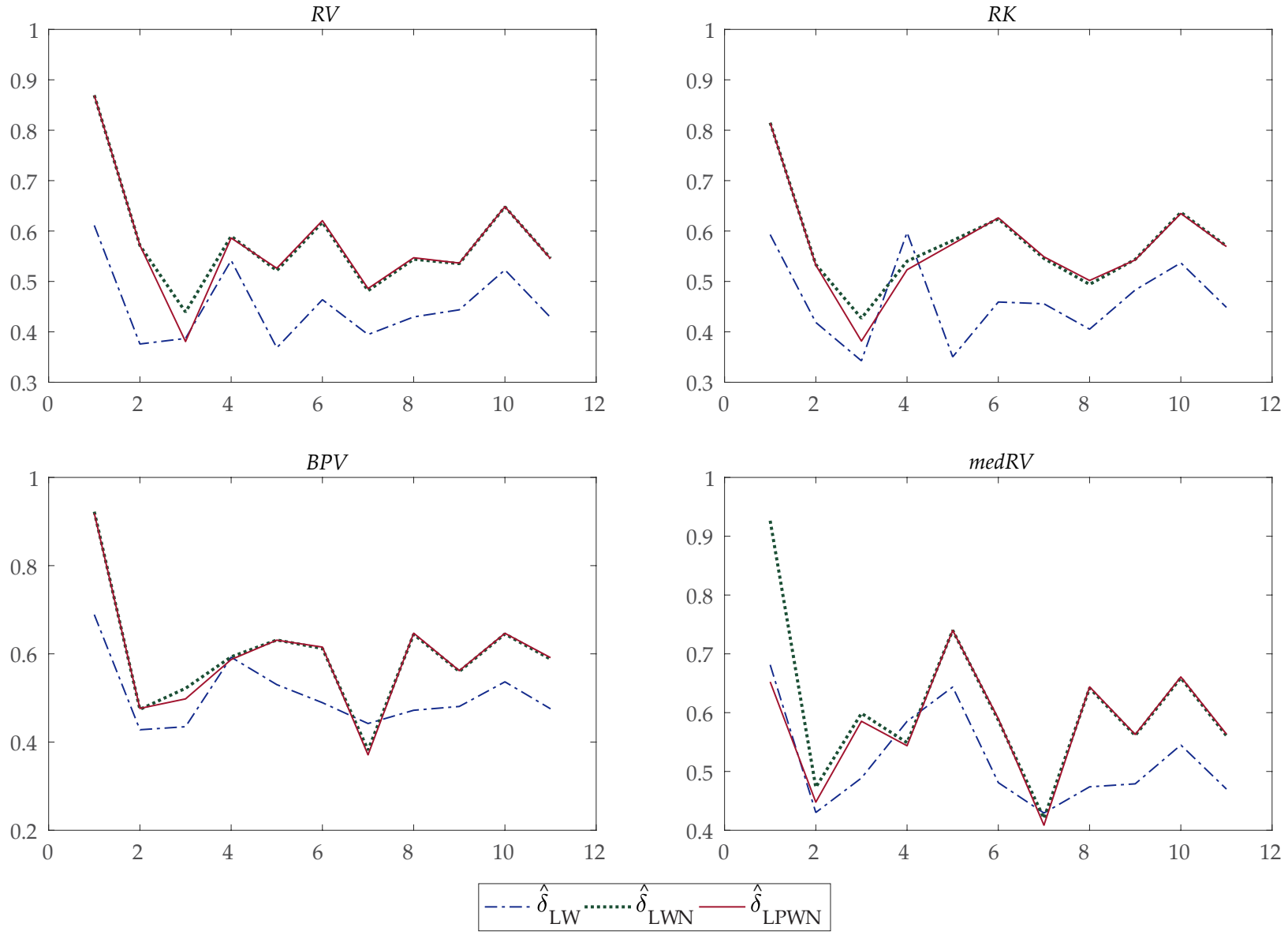
$\hat{a}_u$ obtained from the canonical factorization of the spectral density function

$$\hat{f}_{\log QV}(\tilde{\omega}) = \frac{\hat{\sigma}^2_\varepsilon}{2\pi} \left|1 - e^{-i\tilde{\omega}}\right|^{-2\hat{\delta}} |S(e^{-i\tilde{\omega}}; \hat{\vartheta})|^2,$$

$\tilde{\omega}_j = (2\pi j)/2T,\ j = 0, \ldots, T-1.$

$$\widehat{QV}^f = \exp(\log \widehat{QV}^f + \hat{\sigma}^2_\varepsilon/2)$$

Fractional integration orders



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