Assessing Volatility Persistence in Fractional Heston Models with Self-exciting Jumps

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Volatility dynamics

- complex, unobserved and requires specific methods to model & forecast
- mainly driven by two stylized facts:
  - persistence
  - presence of jumps in asset prices
Volatility dynamics and the theoretical literature

■ Long-range dependence in volatility


■ Volatility modeling in presence of jumps


■ Few empirical papers merge these two stylized facts


■ But no theoretical paper tackles these issues at the same time
Objective

- Propose a realistic continuous-time model, which accounts for both general forms of jumps and persistence
- Study volatility dynamics in this theoretical framework
- Empirically analyze volatility forecasting in this context
Theoretical framework

Continuous-time model:

\[ dS(\tau) = \mu(\mu_S, \lambda^*(\tau), \zeta(\tau))S(\tau)d\tau + \sigma(\tau)S(\tau)dW_S(\tau) + (\zeta_S - 1)S(\tau)dN(\tau) \]

\[ \sigma^2(\tau) = \eta + \int_{-a}^{\tau} \frac{(\tau - s)^\delta}{\Gamma(1 + \delta)}dV(s) \]

\[ dV(\tau) = \kappa(\theta - V(\tau))d\tau + \xi \sqrt{V(\tau)}dW_\sigma(\tau) \]

\[ \lambda^*(\tau) = \bar{\lambda} + \int_{-\infty}^{\tau} \nu(\tau - u)dN(u) \]

with

- \( S(\tau) \) the true price process of a given asset at a continuous time \( \tau \)
- \( \mu \) the compensated drift term
Theoretical framework

Continuous-time model:

\[ dS(\tau) = \mu(\mu_S, \lambda^*(\tau), \zeta(\tau))S(\tau)d\tau + \sigma(\tau)S(\tau)dW_S(\tau) + (\zeta_S - 1)S(\tau)dN(\tau) \]

\[ \sigma^2(\tau) = \eta + \int_{-\infty}^{\tau} \frac{(\tau - s)\delta}{\Gamma(1 + \delta)} dV(s) \]

\[ dV(\tau) = \kappa(\theta - V(\tau))d\tau + \zeta \sqrt{V(\tau)}dW_\sigma(\tau) \]

\[ \lambda^*(\tau) = \bar{\lambda} + \int_{-\infty}^{\tau} v(\tau - u)dN(u) \]

with

- \( \sigma^2(\tau) \) instantaneous fractionally integrated variance process
- \( \delta \in (0, 1/2) \), the long-memory parameter
- \( a \geq 0 \)
- \( W_S(\tau) \) and \( W_\sigma(\tau) \) two independent standard Brownian motions
- Feller condition: \( \theta, \kappa > 0, \kappa \theta \geq \zeta^2 / 2 \)
Theoretical framework

Continuous-time model:

\[
dS(\tau) = \mu(\mu_S, \lambda^*(\tau), \zeta(\tau))S(\tau)d\tau + \sigma(\tau)S(\tau)dW_S(\tau) + (\zeta_S - 1)S(\tau)dN(\tau)
\]

\[
\sigma^2(\tau) = \eta + \int_{-\infty}^{\tau} \frac{(\tau - s)\delta}{\Gamma(1 + \delta)}dV(s)
\]

\[
dV(\tau) = \kappa(\theta - V(\tau))d\tau + \zeta\sqrt{V(\tau)}dW_\sigma(\tau)
\]

\[
\lambda^*(\tau) = \bar{\lambda} + \int_{-\infty}^{\tau} \nu(\tau - u)dN(u)
\]

with

- \((\zeta(\tau) - 1)dN(\tau)\) a finite activity jump process
- \(N(\tau)\) an Hawkes process with time varying intensity parameter \(\lambda^*(\tau)\)
- \(\zeta(\tau)\) i.i.d. log-normal random variable
Quadratic variation of prices

- Solution of the SDE

\[ p(\tau) = p(0) + \int_0^\tau \left( \bar{\mu}(u) - \frac{1}{2} \sigma^2(u) \right) du + \int_0^\tau \sigma(u) dW_S(u) + \int_0^\tau \tilde{\zeta} dN(u), \]

with \( \tilde{\zeta} = \ln (\zeta(u)) \sim \text{i.i.d. } N(\bar{\mu}, \bar{\sigma}). \)

- Quadratic variation

\[
\sum_{j=1}^{M} \left( p(\tau_{t-1} M+j+1) - p(\tau_{t-1} M+j) \right)^2 \xrightarrow{M \to \infty} [p, p]_t = QV(t)
\]

\[
QV(t) = \int_{t-1}^{t} \sigma^2 d\tau + \sum_{N(t)-1}^{N(t)} \tilde{\zeta}(\tau_i)^2
\]

\[ IV_t \equiv C(t) \quad JV_t \equiv D(t) \]
Quadratic variation of prices

Figure: The variance generated by the fractional Heston model with jumps (left) was obtained by setting $\delta = 0.4$, $\kappa = 5$, $\theta = 0.1$, $\zeta = 0.9$, $\eta = 0.3$, $\tilde{\lambda} = 0.001$, $\alpha = 0.8$, $\beta = 1$, $\tilde{\mu} = 0.005$, $\tilde{\sigma} = 0.15$, and $\sigma_\epsilon = 0.01$. The daily realized variance of JPMorgan (right) is based on 5-minutes returns.
Time domain long memory analysis

**ACF decomposition**

\[
ACF(h) = \frac{c_C(h)}{V(C) + V(D)} + \frac{c_D(h)}{V(C) + V(D)}
\]

**Proposition (1)**

Let \( \delta \in (0, 1/2) \). The autocovariance function of \( C(t) \) takes the form

\[
c_C(h) = \int_0^1 \int_0^1 c_{\sigma^2}(u + h - s) \, ds \, du
\]

\[
= \frac{C}{2\delta(2\delta + 1)} \int_{-\infty}^{+\infty} (|v - 1|^{2\delta + 1} + |v + 1|^{2\delta + 1} - 2|v|^{2\delta + 1}) e^{-\kappa|v-h|} \, dv
\]

with \( C = \frac{\zeta^2 \theta}{2\kappa} \frac{\Gamma(1-2\delta)}{\Gamma(1-\delta)\Gamma(\delta)} \), \( h \in [0, \infty) \),

\[
c_{\sigma^2}(h) = C \int_{-\infty}^{+\infty} |h + v|^{2\delta-1} e^{-k|v|} \, dv,
\]

Asymptotically, as \( h \to \infty \)

\[
c_C(h) \sim \frac{2C}{\kappa} h^{2\delta - 1}.
\]
Time domain long memory analysis

Assumption (1)

We suppose that the excitation function is exponential $\nu(t) = \alpha e^{-\beta t}$, with $\alpha \geq 0$, $\beta \geq 0$ and $\alpha / \beta < 1$ so that the intensity process is stationary.

Note: If $\alpha = 0$, $N(t)$ is a standard Poisson point process.

Proposition (2)

Then, the autocovariance function of $\mathcal{D}(t)$ is

$$c_{\mathcal{D}}(h) = \lambda^* (1 - h) \mathbb{V} (\tilde{\zeta}^2) 1_{(h<1)} + \mathbb{E} (\tilde{\zeta}^2) c_{\tilde{N}_t}(h), \text{ with}$$

$$c_{\tilde{N}_t}(h) = (1 - h) \frac{\beta^3 \lambda}{(\beta - \alpha)^3} 1_{(h<1)} + \frac{\alpha \beta \lambda (2\beta - \alpha)}{2(\beta - \alpha)^4} \left(e^{-(\beta-\alpha)(h+1)} + e^{-(\beta-\alpha)(h-1)} - 2e^{-(\beta-\alpha)h}\right),$$
Time domain long memory analysis

Proposition (2, continued)

As a particular case, for $h = 0$, we obtain

$$\mathbb{V}(\mathcal{D}) = \lambda^* \mathbb{E}(\tilde{\zeta}^4) + \lambda^* (\mathbb{E}(\tilde{\zeta}^2))^2 \frac{\alpha (2\beta - \alpha)}{(\beta - \alpha)^2} \left( 1 - \frac{1 - e^{-(\beta - \alpha)}}{\beta - \alpha} \right),$$

where

$$\lambda^* = \bar{\lambda} \beta / (\beta - \alpha)$$

Asymptotically, as $h \to \infty$

$$c_{\mathcal{D}}(h) \sim \mathcal{K} e^{-(\beta - \alpha)h},$$

with

$$\mathcal{K} = \mathbb{E}(\tilde{\zeta}^2)^2 \frac{\alpha \beta \bar{\lambda} (2\beta - \alpha)}{2(\beta - \alpha)^4} \left( e^{-(\beta - \alpha)} + e^{(\beta - \alpha)} - 2 \right).$$
**ACF of QV**

- The function is analytically intractable $\Rightarrow$ numerical approximation

**Figure:** The plots were obtained by setting $\delta = 0.4, \kappa = 5, \theta = 0.1, \varsigma = 0.9, \bar{\lambda} = 0.001, \beta = 1, \tilde{\mu} = 0.005, \tilde{\sigma} = 0.75$ and $\alpha = \{0.8, 0\}$ for the left and right plot, respectively.
Spectral density of $QV$

Proposition (3)

Let $\omega \in [-\pi, \pi]$, $\delta \in (0, 1/2)$ and assumption 1 hold. We prove that

i) For any $\omega$,

$$
f_C(\omega) = \zeta^2 \theta \frac{|1 - e^{-i\omega}|^2}{\omega^{2\delta + 2}(\omega^2 + \kappa^2)}
$$

$$
f_D(\omega) = \lambda^* \frac{|1 - e^{-i\omega}|^2}{\omega^2} \left( \mathbb{E} (\tilde{\zeta}^4) + \mathbb{E} (\tilde{\zeta}^2)^2 \frac{\alpha(2\beta - \alpha)}{\omega^2 + (\beta - \alpha)^2} \right).
$$

ii) As $\omega \to 0$,

$$
f_C(\omega) \sim \varphi \omega^{-2\delta}, \text{ with } \varphi = \zeta^2 \theta / \kappa^2
$$

$$
f_D(\omega) = \lambda^* \left( \text{Var}(\tilde{\zeta}^2) + \mathbb{E}(\tilde{\zeta}^2)^2 \frac{\beta^2}{(\beta - \alpha)^2} \right).
$$
Spectral density of $QV$

It follows that

$$f_{QV}(\omega) = \varsigma^2 \theta \frac{|1 - e^{-i\omega}|^2}{\omega^{2\delta+2}(\omega^2 + \kappa^2)} + \frac{\lambda^*}{\omega^2} \frac{|1 - e^{-i\omega}|^2}{\omega^2} \left( \mathbb{E}(\tilde{\zeta}^4) + \mathbb{E}(\tilde{\zeta}^2)^2 \frac{\alpha(2\beta - \alpha)}{\omega^2 + (\beta - \alpha)^2} \right),$$

and near zero frequency it takes the form

$$f_{QV}(\omega) \sim \omega^{-2\delta} \varphi(\omega) + \kappa(\omega), \quad \omega \to 0,$$

with $\varphi(\omega) = \omega^{2\delta} f_{\mathcal{C}}(\omega)$ and $\kappa(\omega) = f_{\mathcal{D}}(\omega)$.

- In theory $\varphi(\omega)$ and $\kappa(\omega)$ are constant at zero frequency ($\varphi$ and $\kappa$)

- In practice, estimators approximate these terms in the vicinity of the origin and high frequency contamination may occur
Local Whittle-type long memory estimators

- Robust to misspecification of the short-run dynamics

1. Robinson (1995): \( f(\omega) \sim \varphi \omega^{-2\delta} \) as \( \omega \to 0 \), with \( \varphi > 0 \)

\[
\hat{\delta}_{LW} = \arg \min_{\delta \in \Theta} R_m(\delta), \quad \omega_j = 2\pi j / T
\]

where \( R_m(\delta) = \log \hat{\varphi}(\delta) - 2\delta \frac{1}{m} \sum_{j=1}^{m} \log \omega_j, \quad \hat{\varphi}(\delta) = \frac{1}{m} \sum_{j=1}^{m} \omega_j^{2\delta} I(\omega_j) \)

2. Hurvich et al., 2005 (\( \hat{\delta}_{LWN} \)): \( f(\omega) \sim \omega^{-2\delta} \varphi + \kappa \) as \( \omega \to 0 \), with \( \kappa > 0 \)

3. Frederiksen et al. 2008 (\( \hat{\delta}_{LPWN} \)): \( f(\omega) \sim \omega^{-2\delta} \tilde{\varphi}(\omega) + \kappa \)

4. Frederiksen et al. 2012 (\( \hat{\delta}_{LPWLPN} \)): \( f(\omega) \sim \omega^{-2\delta} \tilde{\varphi}(\omega) + \tilde{\kappa}(\omega) \)
DGP

Generate a sequence of discretized log-prices

- Discretization step $\Delta \tau = \tau_j - \tau_{j-1} = 1\text{min} (M = 390)$ and $T = 1000$ days

- Model parameters:
  - Long Memory: $\delta = 0.4$
  - $\sigma^2_{\hat{r}_i}$: $\kappa = 5$, $\theta = 0.1$, $\zeta = 0.9$, $\eta = 0.3$
  - Jump size: $\tilde{\mu} = 0.005$, $\tilde{\sigma} = 0.15$
  - Jump arrival: $\beta = 1$, $(\alpha, \bar{\lambda}) \in \{(0, 0.001), (0.5, 0.0005), (0.8, 0.0002)\}$

  - Poisson J
  - Self-exciting Hawkes J
LW estimators of long memory in IV

Unobserved daily integrated variance: \( IV_t = \sum_{j=1}^{M} \sigma_{(t-1)M+j}^2, t = 1, \ldots, T \)

- 1000 replications
- Model parameters are set to \( \delta = 0.4, \kappa = 5, \theta = 0.1, \zeta = 0.9 \)
- The bandwidth is fixed to \( m = \lfloor T^{0.5} \rfloor \) for \( \hat{\delta}_{LW} \) and to \( m = \lfloor T^{0.8} \rfloor \) for \( \hat{\delta}_{LWN}, \hat{\delta}_{LPWN}, \) and \( \hat{\delta}_{LPWLPN} \)

\[ \Rightarrow \] Performances of all estimators are very close

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Bias</th>
<th>Variance</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\delta}_{LW} )</td>
<td>-0.0030</td>
<td>0.0139</td>
<td>0.0139</td>
</tr>
<tr>
<td>( \hat{\delta}_{LWN} )</td>
<td>0.0444</td>
<td>0.0018</td>
<td>0.0038</td>
</tr>
<tr>
<td>( \hat{\delta}_{LPWN} )</td>
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<td>0.0054</td>
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<tr>
<td>( \hat{\delta}_{LPWLPN} )</td>
<td>0.0448</td>
<td>0.0116</td>
<td>0.0136</td>
</tr>
</tbody>
</table>
LW estimators of long memory in $QV$

- Observed log-prices
  \[ p^*_i = p_{\tau_i} + \varepsilon_{\tau_i}, \]
  with $\varepsilon_{\tau_i} \sim N(0, 0.01)$

- Realized measures of variance ($\hat{QV}$): no optimal sampling frequency

- Measurement error impacts long memory estimates of $\hat{QV}$:

  \underbrace{\text{discretization error} + \text{microstructure noise}} + \text{jumps (+ short-run dynamics)}

  Rossi and Santucci de Magistris (2014)
  (\(\hat{\delta}_{LWN}\) of Hurvich et al. 2005 works well)
## LW estimators of long memory in QV

<table>
<thead>
<tr>
<th></th>
<th>$C_t$</th>
<th>$C_t + D_t$</th>
<th>$C_t$</th>
<th>$C_t + D_t$</th>
<th>$C_t$</th>
<th>$C_t + D_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bias</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$\delta_{LW}$</td>
<td>$\alpha = 0$</td>
<td>$\alpha = 0.5$</td>
<td>$\alpha = 0.8$</td>
<td>$\alpha = 0$</td>
<td>$\alpha = 0.5$</td>
<td>$\alpha = 0.8$</td>
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<tr>
<td>$RV$</td>
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<td>-0.0210</td>
<td>-0.0267</td>
<td>-0.0677</td>
<td>0.0138</td>
<td>0.0135</td>
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<tr>
<td>$BPV$</td>
<td>-0.0181</td>
<td>-0.0172</td>
<td>-0.0200</td>
<td>-0.0570</td>
<td>0.0140</td>
<td>0.0136</td>
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<td>$medRV$</td>
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<td>-0.0199</td>
<td>-0.0570</td>
<td>0.0138</td>
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<tr>
<td><strong>Variance</strong></td>
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<tr>
<td>$\delta_{LWN}$</td>
<td>$\alpha = 0$</td>
<td>$\alpha = 0.5$</td>
<td>$\alpha = 0.8$</td>
<td>$\alpha = 0$</td>
<td>$\alpha = 0.5$</td>
<td>$\alpha = 0.8$</td>
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<tr>
<td>$RV$</td>
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<td>0.0009</td>
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<td>$BPV$</td>
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<td>0.0091</td>
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<td>0.0041</td>
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<td>$medRV$</td>
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<td>-0.0064</td>
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<tr>
<td><strong>MSE</strong></td>
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</tr>
<tr>
<td>$\delta_{LPWN}$</td>
<td>$\alpha = 0$</td>
<td>$\alpha = 0.5$</td>
<td>$\alpha = 0.8$</td>
<td>$\alpha = 0$</td>
<td>$\alpha = 0.5$</td>
<td>$\alpha = 0.8$</td>
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<tr>
<td>$RV$</td>
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<td>-0.0072</td>
<td>-0.0082</td>
<td>-0.0195</td>
<td>0.0063</td>
<td>0.0070</td>
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<td>$BPV$</td>
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<td>-0.0172</td>
<td>0.0069</td>
<td>0.0067</td>
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<tr>
<td>$medRV$</td>
<td>-0.0059</td>
<td>-0.0063</td>
<td>-0.0074</td>
<td>-0.0163</td>
<td>0.0069</td>
<td>0.0068</td>
</tr>
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</table>

Note: The results are based on 1000 replications of $T = 1000$ daily observations based on $M = 390$ intra-daily observations sampled at $\Delta \tau = 1$ minute. The bandwidth is fixed to $m = \lfloor T^{0.5} \rfloor$ for $\delta_{LW}$ and to $m = \lfloor T^{0.8} \rfloor$ for $\delta_{LWN}$ and $\delta_{LPWN}$. $\alpha = 0$ for Poisson jumps and $\alpha > 0$ for Hawkes jumps.
LW estimators of long memory in \( QV \) (model with leverage)

<table>
<thead>
<tr>
<th></th>
<th>Bias</th>
<th>Variance</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( C_t )</td>
<td>( C_t + D_t )</td>
<td>( C_t )</td>
</tr>
<tr>
<td>( \delta_{LW} )</td>
<td>( \alpha = 0 )</td>
<td>( \alpha = 0.5 )</td>
<td>( \alpha = 0.8 )</td>
</tr>
<tr>
<td>( RV )</td>
<td>( -0.0371 )</td>
<td>( -0.0543 )</td>
<td>( -0.0971 )</td>
</tr>
<tr>
<td>( BPV )</td>
<td>( -0.0476 )</td>
<td>( -0.0464 )</td>
<td>( -0.0829 )</td>
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<tr>
<td>( medRV )</td>
<td>( -0.0522 )</td>
<td>( -0.0511 )</td>
<td>( -0.0831 )</td>
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<tr>
<td>( \delta_{LWN} )</td>
<td>( \alpha = 0 )</td>
<td>( \alpha = 0.5 )</td>
<td>( \alpha = 0.8 )</td>
</tr>
<tr>
<td>( RV )</td>
<td>( -0.0197 )</td>
<td>( -0.0210 )</td>
<td>( -0.0137 )</td>
</tr>
<tr>
<td>( BPV )</td>
<td>( -0.0183 )</td>
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<td>( medRV )</td>
<td>( -0.0162 )</td>
<td>( -0.0172 )</td>
<td>( -0.0106 )</td>
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<tr>
<td>( \delta_{LPWN} )</td>
<td>( \alpha = 0 )</td>
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<td>( \alpha = 0.8 )</td>
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<tr>
<td>( RV )</td>
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<td>( -0.0158 )</td>
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<td>( -0.0131 )</td>
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</table>

Note: The results are based on 1000 replications of \( T = 1000 \) daily observations based on \( M = 390 \) intra-daily observations sampled at \( \Delta \tau = 5 \text{ minutes} \). \( \rho = -0.3 \). The bandwidth is fixed to \( m = \lfloor T^{0.5} \rfloor \) for \( \delta_{LW} \) and to \( m = \lfloor T^{0.8} \rfloor \) for \( \delta_{LWN} \) and \( \delta_{LPWN} \). \( \alpha = 0 \) for Poisson jumps and \( \alpha > 0 \) for Hawkes jumps.
A huge literature in finance focuses on risk prediction

- Persistence plays a crucial role
  - GARCH, ARFIMA, HAR

As Baillie et al. (2012), we focus on frequency domain forecasting

1. Filter the long-run component of the log $\hat{QV}$ with $\hat{\delta}_{LW}$ (2S – LW approach) and $\hat{\delta}_{L(P)WN}$ (2S – L(P)WN approaches)

2. Estimate the short-run parameters, $\vartheta$, of each resulting process with ARMA(1, 1) full-band Whittle framework

3. Compute iterated forecasts in frequency domain (see Bhansali and Kokoszka, 2002; Hidalgo and Yajima, 2002)
Data

- The dataset runs from January 2010 to June 2016
- 11 largest US financial institutions
- Rolling forecasting scheme (1000 days in-sample, 635 out-of-sample)
- Forecast horizon $h \in \{1, 5, 10, 22\}$ days ahead
- Use robust loss functions (Patton 2011) to assess out-of-sample forecast accuracy
- ARMA(1,1) and log-HAR as competitors
### QV forecast comparison

**Table:** Variance forecast comparison under various robust loss functions

<table>
<thead>
<tr>
<th></th>
<th>$h = 1$</th>
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<th>$h = 10$</th>
<th>$h = 22$</th>
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<td></td>
<td>MSE</td>
<td>QLIKE</td>
<td>AL</td>
<td>MSE</td>
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<td>RV</td>
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<td>0.8433</td>
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<tr>
<td>KK</td>
<td>1.9424</td>
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<tr>
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<td>2.1895</td>
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<td>2.1654</td>
</tr>
<tr>
<td>BV</td>
<td>8.1588</td>
<td>8.1325</td>
<td>8.1112</td>
<td>8.1074</td>
</tr>
<tr>
<td>medRV</td>
<td>4.9873</td>
<td>4.9623</td>
<td>4.9431</td>
<td>4.9408</td>
</tr>
<tr>
<td>RV</td>
<td>1.1684</td>
<td>1.1321</td>
<td>1.0945</td>
<td>1.0916</td>
</tr>
<tr>
<td>KK</td>
<td>2.3224</td>
<td>2.2872</td>
<td>2.2397</td>
<td>2.2368</td>
</tr>
<tr>
<td>BV</td>
<td>8.3381</td>
<td>8.3293</td>
<td>8.2884</td>
<td>8.2864</td>
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<tr>
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<td>0.4685</td>
<td>0.4174</td>
<td>0.4129</td>
<td>0.4110</td>
</tr>
</tbody>
</table>

Note: The bandwidth is fixed to $m = \lfloor T^{0.5} \rfloor$ for $\hat{\delta}_{LW}$ and to $m = \lfloor T^{0.8} \rfloor$ for $\hat{\delta}_{LWN}$ and $\hat{\delta}_{LPWN}$. 

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G. de Truchis

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Conclusion

- In this paper we have proposed a fractional Heston model with Hawkes-type jump
- We have theoretically derived the autocorrelation and spectral density functions of the QV
- We show that the persistence of QV exhibits a downward bias in presence of jumps
- Robust local Whittle estimators appear as a convenient solution
- We propose a frequency-domain forecasting approach that seems to outperform traditional methods
The end

Thank you for your attention!
LW estimators of long memory in $QV$

Figure: 1-minute log-prices generated by the fractional Heston model with jumps ($p_{t,1}$ and $p_{t,2}$) and the continuous underlying log-price ($p^c_t$). The plots were obtained by setting $\delta = 0.4$, $\kappa = 5$, $\theta = 0.1$, $\zeta = 0.9$, $\eta = 0.3$, $\bar{\lambda} = 0.001$, $\alpha = 0.8$, $\beta = 1$, $\tilde{\mu} = 0.005$, and $\sigma_\epsilon = 0.01$. $\tilde{\sigma} = 0.15$ for $p_{t,1}$ and $\tilde{\sigma} = 0.5$ for $p_{t,2}$. 
Our frequency-domain forecasting approach for $QV$

3. One-step and multistep forecasts

$$\log \widehat{QV}^f_{T+1} = - \sum_{u=1}^{M-1} \hat{a}_u \log \widehat{QV}_{T+1-u},$$

$$\log \widehat{QV}^f_{T+h} = - \sum_{u=1}^{h} \hat{a}_u \log \widehat{QV}^f_{T+h-u} - \sum_{u=1}^{M-h-1} \hat{a}_{u+h} \log \widehat{QV}_{T-u},$$

with $M > h$ the optimal number of lags (AIC with penalty $v = 1.2$) and $\hat{a}_u$ obtained from the canonical factorization of the spectral density function

$$\hat{f}_{\log QV}(\tilde{\omega}) = \hat{\sigma}^2 \frac{1 - e^{-i\tilde{\omega}}}{2\pi} |1 - e^{-i\tilde{\omega}}|^{-2\delta} |S(e^{-i\tilde{\omega}}; \hat{\theta})|^2,$$

$$\tilde{\omega}_j = (2\pi j)/2T, \ j = 0, \ldots, T - 1.$$
Fractional integration orders


